Meriam 3/1 – 3/5 : Kinetics of Particles

- $\sum \vec{F} = m\vec{a}$
  - vector equation
  - valid in all inertial reference frames & coord systems
  - use a free body diagram to solve

- Common Forces:

gravity force $= \dfrac{G m_1 m_2}{r^2}$

$= mg$ on surface of planet

spring force $= -kx$

friction force

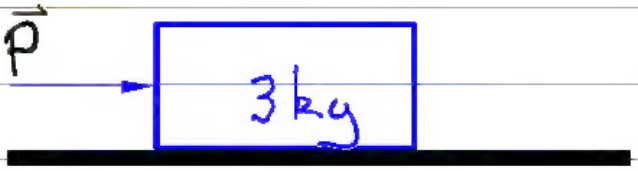
static: $F \leq \mu_s N$

kinetic: $F = \mu_k N$

Friction example

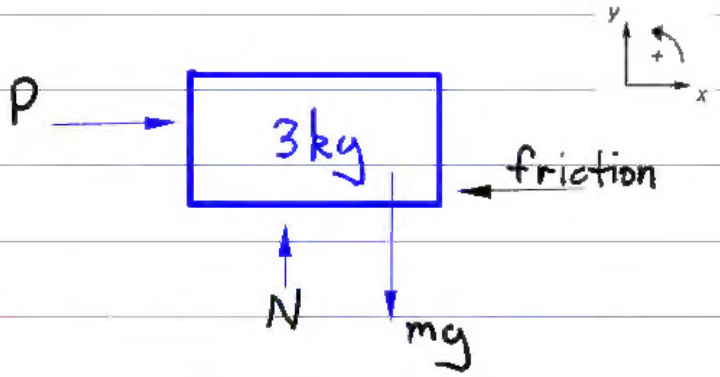
$\mu_s = 0.3$

$\mu_k = 0.25$



A block is sitting on the ground. I increase $\vec{P}$ from zero. How will the block behave?

Answer



1/ Draw FBD

2/ Select coordinate system

3/ $\Sigma F_y = ma_y = 0$

$\rightarrow N - mg = 0$

$N = mg = (3)(9.81) = 29.43$ Newtons

4/ System is originally static. Friction will keep it static until the static friction force reaches $\mu_s N$

$$(\text{Friction}_s)_{max} = \mu_s N$$
$$= (0.3)(29.43) = 8.83 \text{ Newtons}$$

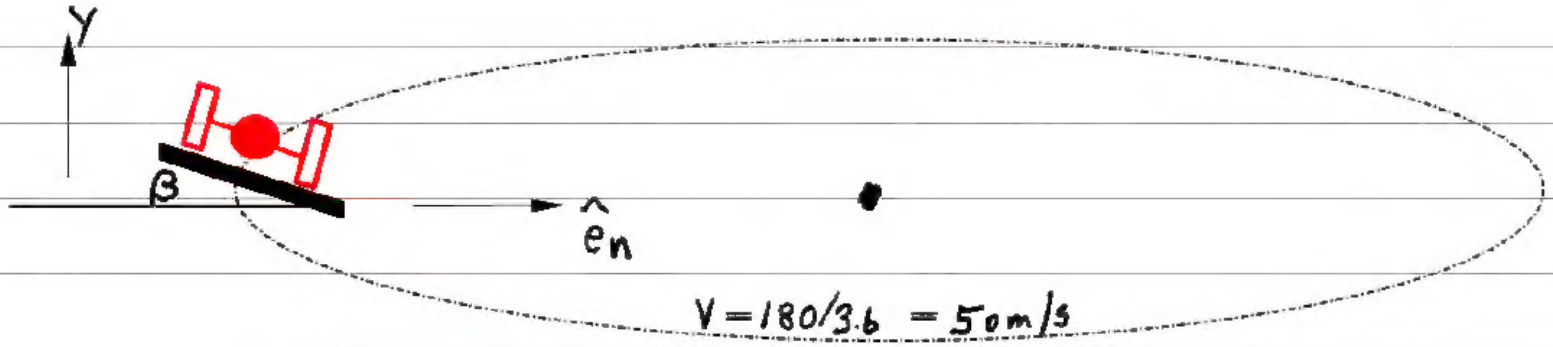
→ Block will remain stationary, with friction force increasing to counterbalance P, until P reaches 8.83 Newtons

5/ Once P reaches 8.83 Newtons, block will slide
→ friction force becomes kinetic friction

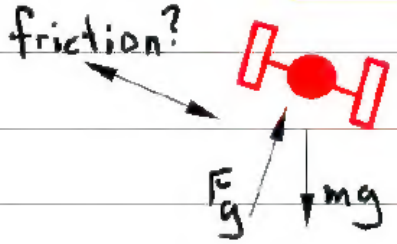
$$\sum F_x = m a_x$$
$$P - \mu_k N = m a_x$$
$$8.83 - (0.25)(29.43) = 3 a_x$$
$$\boxed{a_x = 0.49 \ m/s^2}$$

Example

A circular race track has radius R = 200 m. Determine the track's bank angle β if rated speed is V = 180 km/hr.



FBD



1/ $\sum F_n = m a_n$

$F_g \sin\beta = m v^2 / R$

$$\rightarrow \boxed{F_g = \frac{m v^2 / R}{\sin\beta}} \quad (1)$$

2/ $\sum F_y = 0 = -mg + F_g \cos\beta$

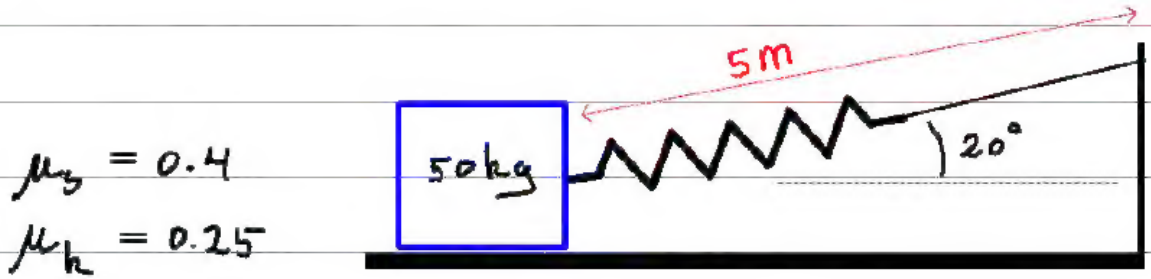
$$\rightarrow \boxed{F_g = mg / \cos\beta} \quad (2)$$

Divide (2) by (1)

$$\rightarrow 1 = \left(\frac{mgR}{mv^2}\right)\left(\frac{\sin\beta}{\cos\beta}\right) \quad (\sin\beta/\cos\beta = \tan\beta)$$

$$\rightarrow \boxed{\beta = 51.9^\circ}$$

Eg. A spring's relaxed length is 3m.
The block starts to slide only if spring length ≥ 5 m

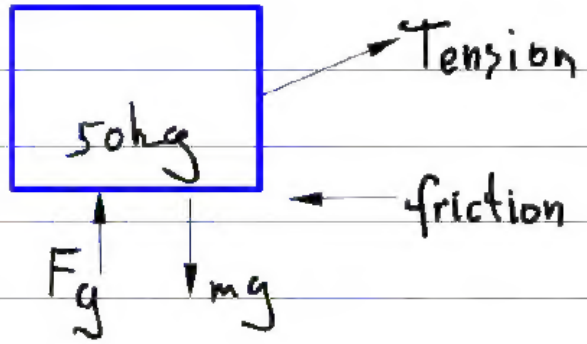


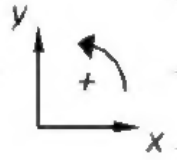
$\mu_s = 0.4$
$\mu_k = 0.25$

Find k for spring

Answer

FBD





Note: $F_g \neq mg$
friction $= \mu_s F_g$ when spring length = 5 m

① $\sum F_y = 0 = F_g - mg + T \sin 20°$

② $\sum F_x = 0 = T \cos 20° - \mu_s F_g$

} 2 unknowns

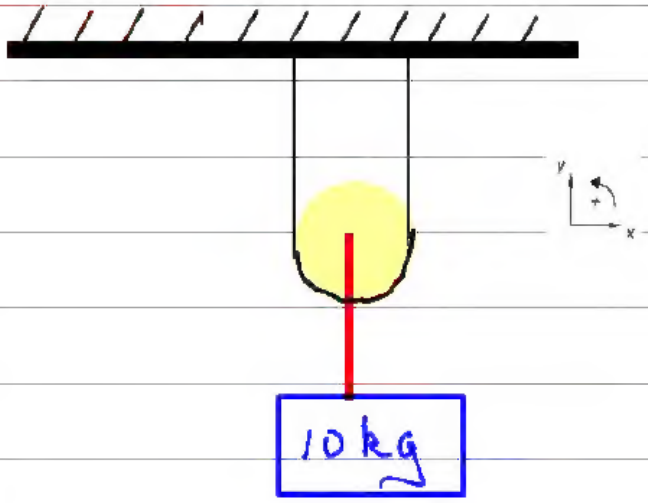
→ $T = 182$ Newtons ; $F_g = 428$ Newtons

but spring tension = (k)(spring extension)

182 Newtons = (5m − 3m) k → **k = 91 Newtons/meter**

Newton's Law with Ropes & Pulleys

Find tension T in rope



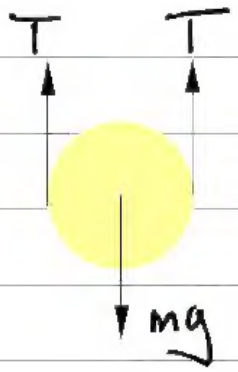
Assume:

1/ Rope does not stretch, nor bunch up.

2/ Rope has zero mass

3/ Tension in the rope is the same everywhere

FBD of wheel:



$$\Sigma F_y = 0$$

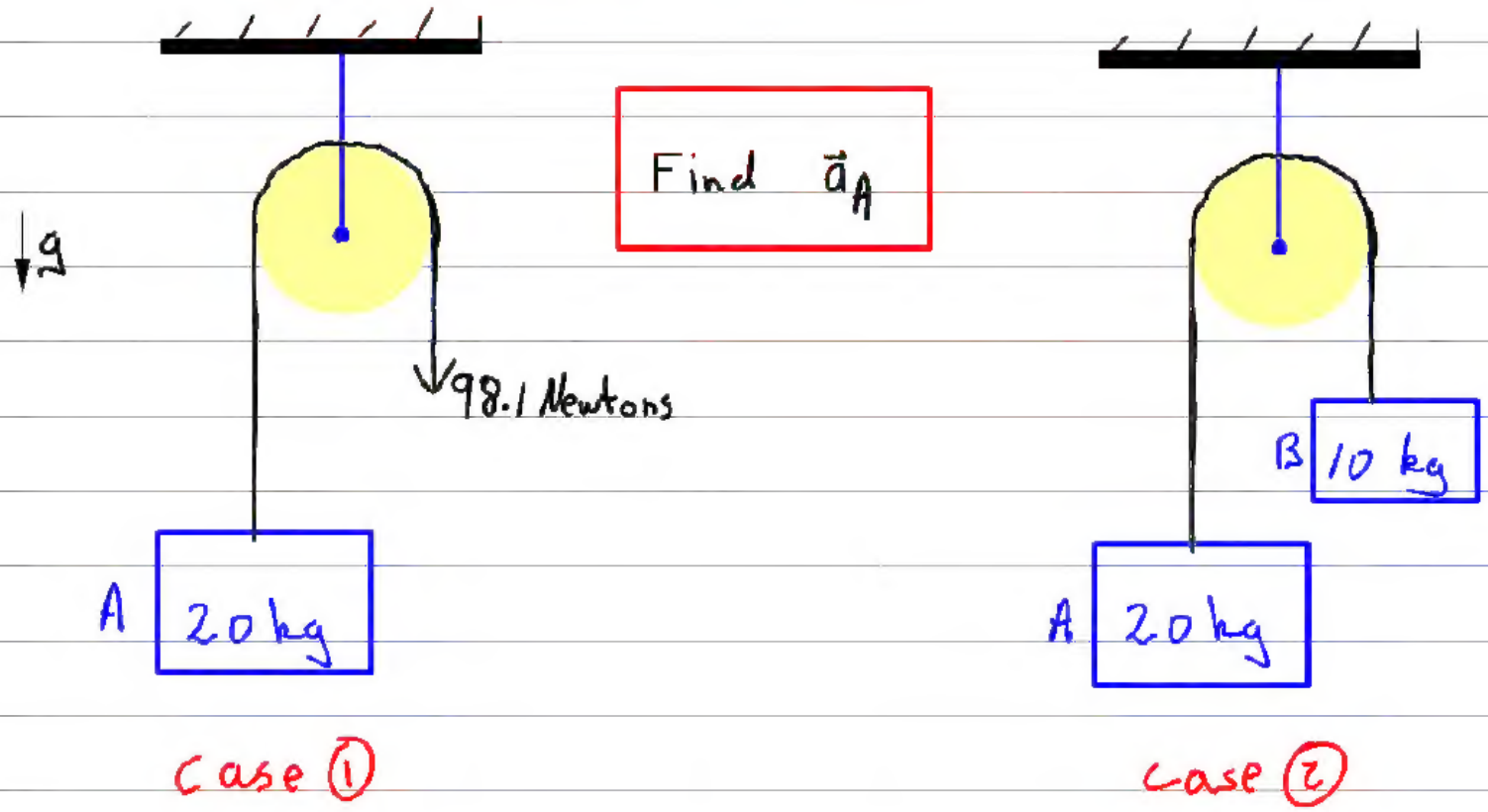
$$T + T - mg = 0$$

$$T = mg/2 = (10)(9.81)/2 = 49 \text{ Newtons}$$

Now make the problem more interesting ...

10 (F)

Compare these 2 cases



Case (1)

Case (2)

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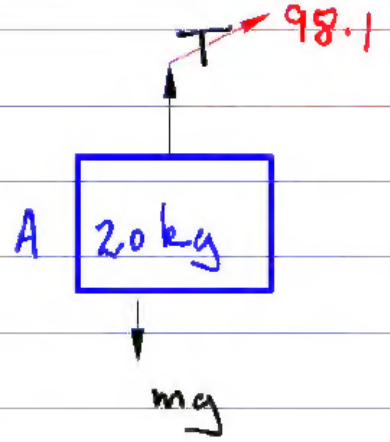
(1)

FBD



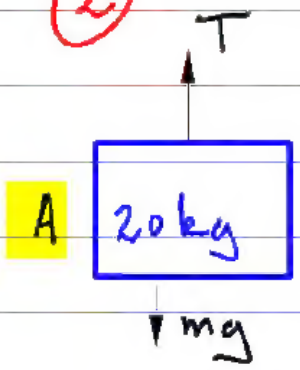
$+\downarrow$ $\quad \Sigma F_A = m a_A$

$(20)(9.81) - 98.1 = 20\, a_A$

$\vec{a}_A = 4.9 \text{ m/s}^2 \downarrow$ for case (1)

$= 4.9 \hat{j} \text{ m/s}^2$

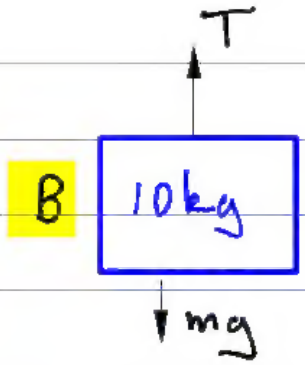
(2)



$\Sigma F_A = m_A a_A \rightarrow$

$(20)(9.81) - T = 20\, a_A$

+ y



$\Sigma F_B = m_B a_B \rightarrow$

$(10)(9.81) - T = 10\, a_B$

Watch +/- signs

2 equations, but 3 unknowns: T, a_A, a_B

There is constrained motion $\rightarrow$ $a_A = -a_B$

$\rightarrow$

$T = 131\ N$

$\vec{a}_A = 3.27\ \hat{j}\ m/s^2$

$\vec{a}_B = -3.27\ \hat{j}\ m/s^2$

case (2)

Why is $|\vec{a}_A|$ lower in case (2) than case (1)?

Summary

- Engineering examples of $\vec{F} = m\vec{a}$
- Static & Dynamic Friction
- For problems with multiple masses, use kinematic relationships among the masses

Next Lecture

- More advanced engineering applications of particle kinetics.
- Meriam 3/1 – 3/5